Algebra I Standards and Learning Activities

Algebra I

NUMBER SENSE AND OPERATIONS INDICATORS

Note: Standards A1.N.1-4 constitute a review of arithmetic material that should be familiar already from the earlier grades.

AI.N.1. Use the properties of operations on real numbers, including the associative, commutative, identity, and distributive properties, and use them to simplify calculations.

Example: Write an equation that matches the following description of a calculation:
You'll get $\frac{1}{8}$ if you start with 3, add 13, divide by 2 and then take the reciprocal.

Example: Simplify the following expression: $2 \bullet \left(\dfrac{3 \bullet 2 + 6}{4} \right) - 4 \div 2$

AI.N.2. Simplify numerical expressions, including those involving integer exponents or the absolute value (e.g., $3(2^4 - 1) = 45$, $4|3 - 5| + 6 = 14$); apply such simplifications in the solution of problems.

Example: Simplify the following expression: $2 \bullet \dfrac{|3 - 5|}{(3 - 5)^2}$

AI.N.3. Calculate and apply ratios, proportions, rates, and percentages to solve a range of consumer and practical problems.

Example: Your current job pays $770 per week. What will your new pay be if you receive a 4% raise?

Example: Airlines know from experience that 3% of the tickets sold for a flight will be "no shows." How many tickets should the company sell if it wants to fill every seat on a plane that has 130 seats?

Example: The current model for a rectangular television screen is 20 inches wide and 15 inches high. The new model is advertised as having an area that is 15% larger than the old model. Assume that the ratio of width to height remains the same and find the dimensions for the new model.

AI.N.4. Use estimation to judge the reasonableness of results of computations and of solutions to problems involving real numbers, including approximate error in measurement and the approximate value of square roots. (Reminder: This is without the use of calculators.)

Example: Estimate. $\sqrt{3} \approx 1.7$

Example: Explain how to use the fact that $\sqrt{3}$ is approximately 1.7 to show that $\sqrt{48}$ is approximately 6.8. ($\sqrt{48} \approx \sqrt{4^2 \cdot 3}$)

AI.N.5. Understand the concept of *nth* roots of positive real numbers and of raising a positive real number to a fractional power. Use the rules of exponents also for fractional exponents.

AI.N.6. Apply the set operations of union and intersection and the concept of complement, universal set, and disjoint sets, and use them to solve problems, including those involving Venn diagrams.

Example: There are 30 students in the class. Twenty of these are taking AP Calculus and 15 are taking AP Physics. If 5 students are taking both AP Calculus and AP Physics, how many students in the class are taking neither *of these AP classes?*

Example: Think about the number of students who plan to go to college **or** *are on the basketball team and the number of students who plan to go to college* **and** *are on the basketball team. Which number is larger? Explain why.*

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS

AI.P.1. Recognize, describe, and extend patterns governed by a linear, quadratic, or exponential functional relationship or by a simple iterative process (e.g., the Fibonacci sequence).

Example: Consider a bunch of rectangles with area equal to 36 square inches. The width (W) of the rectangles varies in relation to the length (L) according to the formula W = 36/L. Make a table showing the widths for all possible whole-number lengths for rectangles with $4 \leq L \leq 36$. Look at the table and examine the pattern of the differences between consecutive entries for L and W. As the length increases by 1, the width decreases but not a constant rate. Sketch the graph of the relationship between L and W. Will the graph be a straight line? Why or why not?

(See also AI.P.4, AI.P.5)

AI.P.2. Use properties of the real number system to judge the validity of equations and inequalities and to justify every step in a sequential argument.

AI.P.3. Demonstrate an understanding of relations and functions. Identify the domain, range, and dependent and independent variables of functions.

Example: The graph of the function $f(x) = x^2 - 2x + 2$ is a parabola that opens up and has vertex at the point (1, 1). What is the range for the function?

(See also AI.P.4)

Example: Find the domain for the function $g(x) = \sqrt{2x - x^2 + 2}$

(See also AI.P.4, AI.P.12, AI.P.13)

AI.P.4. Translate between different representations of functions and relations: graphs, equations, sets of ordered pairs (scatter plots), verbal, and tabular.

Example: The following table records a relation between the width and length of a rectangle with area 36 square inches. Plot the given points and sketch the graph of the relationship between L and W.

W	2	4	6	8	10
L	18	9	6	4.5	3.6

AI.P.5. Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and *x*- and *y*-intercepts from its graph or from a linear equation that represents the line.

Example: What is the slope of the line with equation $3x + 4y = 12$.

Example: Find the equation for the line that contains the points (5, 3) and (7, 4). Where does the line intersect the y-axis? What is the slope of the line?

AI.P.6. Find a linear function describing a line from a graph or a geometric description of the line (e.g., by using the point-slope or slope *y*-intercept formulas). Explain the significance of a positive, negative, zero, or undefined slope.

Example: What kind of line has slope equal to 0?

Example: Is it good news if your math teacher says that your homework grades show a positive slope? Explain why or why not.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS (CONTINUED)
AI.P.7. Find linear functions that represent lines either perpendicular or parallel to a given line and through a point (e.g., by using the point-slope form of the equation).
Example: Find an equation for the line through the origin which is parallel to the line $3x + 4y = 1$.
Example: Lines that are tangent to a circle are perpendicular to the radius of the circle. Find the equation for the line through the point (4,3), which is tangent to the circle of radius 5 centered at the origin.
AI.P.8. Add, subtract, and multiply polynomials with emphasis on 1st- and 2nd-degree polynomials.
Example: Simplify the following expression: $(3x + 1)(x - 2) + (4x + 1)$.
AI.P.9. Demonstrate facility in symbolic manipulation of polynomial and rational expressions by rearranging and collecting terms, factoring [e.g., $a^2 - b^2 = (a + b)(a - b)$, $x^2 + 10x + 21 = (x + 3)(x + 7)$, $5x^4 + 10x^3 - 5x^2 = 5x^2(x^2 + 2x - 1)$], identifying and canceling common factors in rational expressions, and applying the properties of positive integer exponents.
AI.P.10. Divide polynomials by monomials with emphasis on 1st- and 2nd-degree polynomials.
Example: Simplify the following expression: $\dfrac{x^2+3x+2}{x+1}$
Example: Divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.
AI.P.11. Perform basic arithmetic operations with rational expressions and functions.
AI.P.12. Find solutions to quadratic equations (with real roots) by factoring, completing the square, or using the quadratic formula. Demonstrate an understanding of the equivalence of the methods
Example: Solve $x^2 - 3x + 2 = 0$
Example: Use the quadratic formula to explain why the equation $ax^2 + bx + c = 0$ has two, distinct solutions if $b^2 - 4ac > 0$.
Example: Find a quadratic equation that has roots $x = -2$ and $x = +3$.
Example: At how many points does the graph of $y = 2x^2 - x + 1$ intersect the x-axis?
AI.P.13. Solve equations and inequalities, including those involving absolute value of linear expressions (e.g., $\|x - 2\| > 5$), and apply to the solution of problems.
Example: Solve for x: $5x - 2 \leq -3(x - 2) + x$.
AI.P.14. Solve everyday problems (e.g., compound interest and direct and inverse variation problems) that can be modeled using linear or quadratic functions. Apply appropriate graphical or symbolic methods to the solution.
Example: One business telephone service has a fixed monthly cost of $3 per month and then 4 cents per minute for long-distance calls. A second service has no fixed monthly cost but the long-distance calls cost 16 cents per minute. Which service is a better choice? When? (The monthly costs are equal if the company uses 2,500 minutes each month.)
Example: A train travels at 30 miles per hour for one mile. How fast must the train go in the next mile in order to average 60 miles per hour for the full two miles? (Note: This is a tricky problem.)

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS (CONTINUED)

AI.P.15. Solve everyday problems (e.g., mixture, rate, and work problems) that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution.

Example: Mary drove to work on Monday at 40 mph and arrived 5 minutes late. She left at the same time on Friday, drove at 45 mph, and arrived 3 minutes early. How far does Mary drive to work?

Example: Amtrak sells two types of tickets for train service between Boston and Washington, D.C. Tickets for the (really fast) Acela Express sell for $176. Tickets for the (really slow) regular train sell for $91. How many of each type of ticket must Amtrak sell each day if the net revenue for the day must by at least $44,750? What if you add the constraint that the company must sell at least twice as many regular tickets as Acela tickets?

Example: Sketch a graph of the values of x *and* y *that satisfy both of the following inequalities:* $3x + 2y \geq 3$ *and* $-2x + y \geq 5$.

DATA ANALYSIS, STATISTICS, AND PROBABILITY INDICATORS

AI.D.1. Select, create, and interpret an appropriate graphical representation (e.g., scatter plot, table, stem-and-leaf plots, circle graph, line graph, and line plot) for a set of data, and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these notions to compare different sets of data.

Example: According to the 1990 U.S. Census, 27.2% of State X residents over the age of 25 had graduated from a 4-year college. In a circle graph representing all state residents over the age of 25, about how many degrees should be in the sector representing these 4-year college graduates?

Example: The math teacher wants to show his class their grades on a test. Here is a list of the scores:

50, 54, 70, 70, 72, 72, 72, 76, 80, 81, 86, 86, 90, 90, 92, 95, 100

Which of the following types of graphs would give the best picture of the data: scatter plot, stem-and-leaf plot, or a line plot? Pick your favorite and make it.

What is the median score for the class? What is the mean score?

Example: A class of 25 students is asked to determine approximately how much time the average student spends on homework during a one-week period. Each student is to ask one of his/her friends for the information, making sure that no one student is asked more than once. The number of hours spent on homework per week are as follows:

8, 0, 25, 9, 4, 19, 25, 9, 9, 8, 0, 8, 25, 9, 8, 7, 8, 3, 7, 8, 5,
3, 25, 8, 10

(a) Find the mean, median, and mode for these data. Explain or show how you found each answer.

(b) Based on this sample, which measure (or measures) that you found in part (a) best describes the typical student? Explain your reasoning.

(c) Describe a sampling procedure that would have led to more representative data.